

THERMALIZATION

IN QCD

R. Baier

A.H. Mueller

D. Schiff

D.T.S.

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CONCLUSION

- Thermalization occurs in perturbative QCD

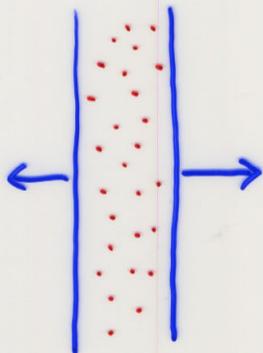
$$\begin{array}{l} \text{thermalization time} \sim \frac{1}{\alpha^{13/5} Q_s} \quad \downarrow \\ \text{temperature} \sim \alpha^{2/5} Q_s \quad \nearrow \end{array} \quad \text{as } Q_s \nearrow$$

Importance of inelastic processes

2 → 2 DOES NOT EFFICIENTLY THERMALIZE

Consider the regime of one-dimensional expansion

$$t \ll R_{\text{nucl}} \sim A^{1/3} \text{ fm}$$



When $t \sim Q_s^{-1}$: gluons are produced, $p \sim Q_s$,
occupation number $\sim 1/\alpha \Rightarrow N \sim Q_s^3/\alpha$

Subsequently:

$$N(t) = \frac{Q_s^3}{\alpha(Q_s t)}$$

Mean free time:

$$\tau = \frac{1}{\sigma N} = \frac{Q_s^2}{\alpha^2} \frac{\alpha Q_s t}{Q_s^3} = \frac{t}{\alpha}$$

or: thermalization time \gg Hubble time.

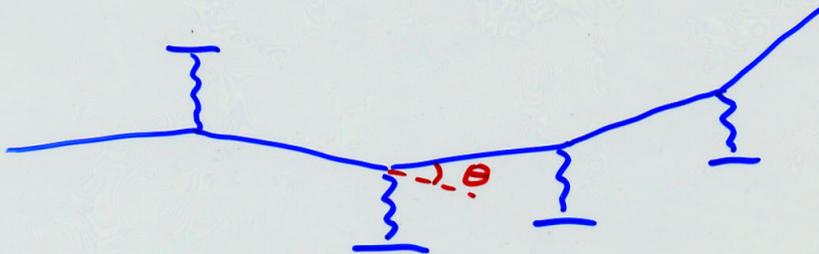
worse at higher collision energy

WHY $2 \rightarrow 2$ IS INEFFICIENT

A lot of small angle scattering: $\sigma = \frac{\alpha^2}{q^2}$

q may be $\ll Q_s$, σ seems very large

but each deflects the particle only by an angle $\theta \sim \frac{q}{Q_s}$.

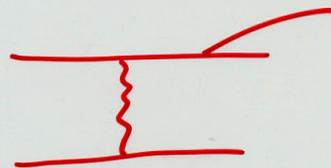


Needs $\frac{1}{\theta^2} \sim \frac{Q_s^2}{q^2}$ small angle scatterings to change the distribution function \Rightarrow relaxation is determined by the transport mean free time, $\sigma_{tr} = \frac{\alpha^2}{q^2} \frac{q^2}{Q_s^2} = \frac{\alpha^2}{Q_s^2}$.

IMPORTANCE OF $2 \rightarrow 3$

$\sigma_{2 \rightarrow 3} \sim \alpha \sigma_{2 \rightarrow 2}$, but does **not** requires multiple scatterings to change the distribution function \Rightarrow very efficient:

$$\sigma_{2 \rightarrow 3} \sim \frac{\alpha^3}{m_D^2}$$



m_D = Debye screening, coming from primary hard gluons and secondary soft gluons emitted from $2 \rightarrow 3$

Timeline

$\frac{1}{\Lambda_s}$

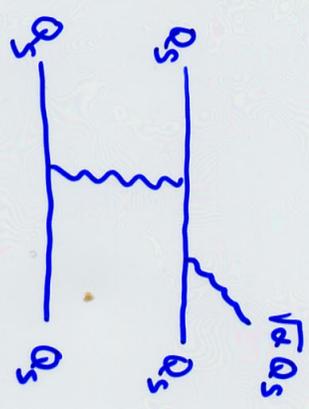
gluons become on shell

$\frac{1}{\alpha} \gg \beta \gg 1$

classical field theory
kinetic theory

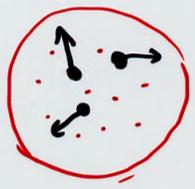
$\frac{1}{\Lambda_s} \alpha^{3/2} \Lambda_s$

accumulation of soft ($\sqrt{\alpha} \Lambda_s$) gluons



$\frac{1}{\Lambda_s} \alpha^{5/2} \Lambda_s$

Thermalized soft sector + hard particles



$\frac{1}{\Lambda_s} \alpha^{13/5} \Lambda_s$

complete thermalization

$LPM \sim \frac{1}{\alpha}$

$T \sim \alpha^{1/2} \Lambda_s$

$T \sim \alpha^{2/5} \Lambda_s$

LPM

$\frac{N_{final}}{N_{in}} \sim \frac{1}{\alpha^{2/5}}$

From classical field eq to kinetic Boltzmann eq.

A.H. Mueller, DTS in progress

Classical field equation (schematically)

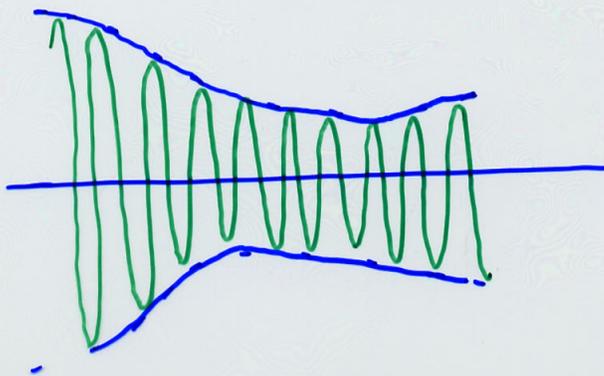
$$\partial^2 A + gA\partial A + g^2 A^3 = 0$$

when $\tau \sim Q_s^{-1}$ $A \sim \frac{1}{g}$: strong nonlinearity

expansion $\Rightarrow A \downarrow$ $A \ll \frac{1}{g}$ when $\tau \gg Q_s^{-1}$

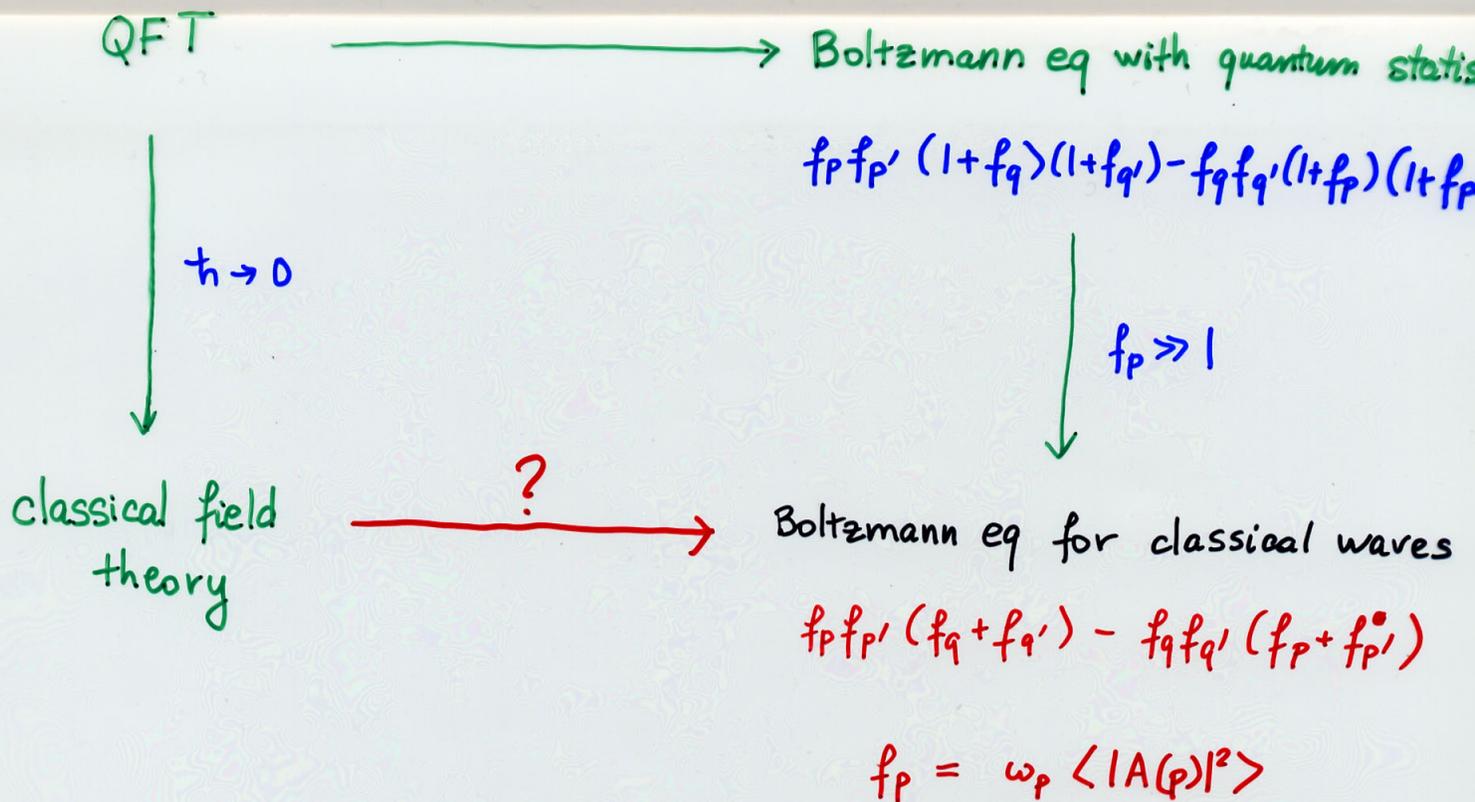
Regime of weak nonlinearity

$$A(t, \vec{x}) = \int d\vec{p} \underbrace{A(t, \vec{p})}_{\text{slow variation in } t} e^{-i\omega_p t + i\vec{p} \cdot \vec{x}} + \text{H.c.}$$



Eq for envelope : Boltzmann eq

Indirect way to derive Boltzmann eq



Is there a way to directly go from classical field eqns to the Boltzmann eqn?

One method: solve classical field eqn perturbatively
identify secular terms growing with time

does not have the generality one would like

The Martin - Siggia - Rose formalism

Martin, Siggia, Rose PRA, 8, 423 (1973)

Consider ϕ^4 theory

$$\partial_\mu^2 \phi + m^2 \phi + \lambda \phi^3 = 0$$

Generating functional:

$$Z = \int \mathcal{D}\phi \mathcal{D}\pi \exp \left\{ -i \int d^4x \left[\pi (\partial^2 \phi + m^2 \phi + \lambda \phi^3) + J_\pi \pi + J_\phi \phi \right] \right\}$$

↑
enforce field eq

field doubling: classical limit of Schwinger - Keldysh

⇒ Feynman rules:



$$G_{\phi\pi}(x, y) = \langle \phi(x) \pi(y) \rangle$$

$$= G_R(x, y) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ipx}}{p^2 - m^2 + i\epsilon \operatorname{sgn} p_0}$$



$$G_{\pi\phi}(x, y) = G_A(x, y)$$



$$G_{\phi\phi}(x, y) = \langle \phi(x) \phi(y) \rangle$$



$$-i\lambda$$

Schwinger - Dyson eqn :

A red line representing a propagator is shown with an equals sign. To the right, there are two terms: the first is a red line with a red circle containing a dot in the middle, representing a self-energy correction; the second is a red line with a red wavy line (representing a pion) attached to it, also with a red circle containing a dot in the middle, representing another type of self-energy correction.

$$\left\{ \begin{aligned} (\partial_x^2 + m^2) G_{\phi\phi}(x, y) &= \int dz \left[\Sigma_{\pi\phi}(x, z) G_{\phi\phi}(z, y) + \Sigma_{\pi\pi}(x, z) G_{\pi\phi}(z, y) \right] \\ (\partial_y^2 + m^2) G_{\phi\phi}(x, y) &= \int dz \left[G_{\phi\phi}(x, z) \Sigma_{\phi\pi}(z, y) + G_{\phi\pi}(x, z) \Sigma_{\pi\pi}(z, y) \right] \end{aligned} \right.$$

and Wigner transform

$$G(x, y) = \int dp e^{-ip \cdot (x-y)} G\left(\frac{x+y}{2}, p\right)$$

assume slow variation in $X = \frac{x+y}{2}$, fast in $x-y$

(separation of scales)

$$p^\mu \frac{\partial}{\partial X^\mu} G_\pi(X, p) = \underbrace{G_{\phi\phi} (\Sigma_{\pi\phi} - \Sigma_{\phi\pi}) + \Sigma_{\pi\pi} (G_{\pi\phi} - G_{\phi\pi})}_{\text{collision term}}$$

collision term

with correct statistical factor

PERTURBATIVENESS

Heavy ion collisions at ^{very} high energy: can be described perturbative QCD.

$$Q_s \sim \begin{cases} 1 \text{ GeV at RHIC} \\ 2 - 3 \text{ GeV at LHC} \end{cases}$$

Q: does the system thermalize?

Pro: more gluons produced, collide more frequently.

Con:

- gluon distribution initially far from equilibrium
- α_s smaller at higher energies

Goal: to have a consistent understanding of evolution when $\alpha_s \ll 1$, which is valid parametrically.